

# DOMINATED SPLITTINGS, THERMODYNAMICAL FORMALISM AND (HIGHER RANK) GEOMETRIC STRUCTURES

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In the recent years the field of higher rank Teichmüller theory has seen a surprisingly fast and deep development. Inspired by classical Teichmüller Theory and being at the intersection of several fields as Geometric Group Theory, Geometry of Symmetric spaces, Dynamical Systems, among others, it is a natural place find beautiful interactions between these.

We will begin by relating dominated splittings of linear cocycles with higher rank Morse lemmas and, particularly, a class of discrete subgroups of  $\mathrm{SL}(n, \mathbb{K})$  for  $\mathbb{K} = \mathbb{R}$  or  $\mathbb{C}$ , called Anosov representations. These representations were introduced by Labourie in 2006 and gather together different classes treated independently up to that date such convex-co-compact subgroups of rank 1 symmetric spaces, strictly convex projective structures, higher rank Schottky groups.

We will then study the moduli space of these representations and build a natural Riemannian metric on it, called *the Pressure metric*. When one deals with the classical Teichmüller space, this dynamical construction provides a multiple of Weyl-Peterson metric, defined purely in complex-analytic terms.

We will end by studying applications of the above, to critical exponents in the Hitchin component (the first appearance of a *higher rank Teichmüller space*), and/or the study of Hausdorff dimension of limit sets of discrete subgroups of the complex-hyperbolic space.

The material covered here can be found in the following references: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]

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